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INVERSE DETERMINATION OF TEMPERATURE IN DUCTS WITH H1 BOUNDARY CONDITIONS

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INTRODUCTION

For flow in different passages with a constant axial heat flux per unit length, two types of boundary conditions are classified in the literature: one is called H1 boundary condition and the other one is H2 boundary condition. The H2 case considers the local heat flux to have a constant value and it has a direct solution. The H1 boundary condition considers the heat flux per unit length to be uniform while the wall temperature remains a function of the axial coordinate alone. Unlike the H2 boundary condition, the H1 does not have a direct solution in the thermally developing region. For H1 boundary condition, the inverse methodology is a useful tool for determination of thermal phenomena in ducts with different cross-section profiles. Solutions are desirable for small and large values of axial coordinate. A unified solution methodology is used at small values of axial coordinate and it has a broad range of applications to ducts having different cross section profiles. The function specification method becomes a useful tool for inverse determination of the wall temperature at relatively large values of axial coordinate.

The inverse analysis data are summarized into the following correlation for rectangular passages with $2a \times 2b$ cross section dimensions

$$
\begin{aligned}
Nu_D = \frac{hD_h}{k} &= \frac{0.651\, D_h}{(\hat{x}/C)^{1/3} + (0.58/\bar{b})(\hat{x}/C)^{2/3}} \\
&\quad + \frac{\mathrm{Nu}_{D,FD}}{\left[1 + \left(2.48 - \dfrac{0.24}{\bar{b}^2}\right)(C^2\ \hat{x})^{-2/3}\right]^{4/3}} \quad \text{for } 0 \le \hat{x} \le 0.8 \\
&= Nu_{D,FD} \quad \text{for } 0.8 < \hat{x} < \infty
\end{aligned}
\tag{1}
$$

where D_h is the hydraulic diameter, h is circumferentially averaged heat transfer coefficient, k is the fluid thermal conductivity, $\bar{b} = b/a$ is the aspect ratio, and $\hat{x} = (x/a)/Pe$ while $Pe = Ua/\alpha$ and α is the thermal diffusivity. The average velocity slope at the wall is C=3.325, 2.731, 2.689, 2.739, 2.837, 3.0 for aspect ratios $\bar{b} = b/a$ = 1, 2, 3, 5, 10, ∞, respectively. For the same aspect ratios, the Nusselt numbers for thermally fully developed condition are $Nu_{D,FD}$ = 3.608, 3.092, 3.196, 3.442, 3.732, 4.118 and they are obtained through direct solution using Fourier series. The computed data using the small-x solution and the function specification method are compared with the above correlation and the results are graphically presented below.

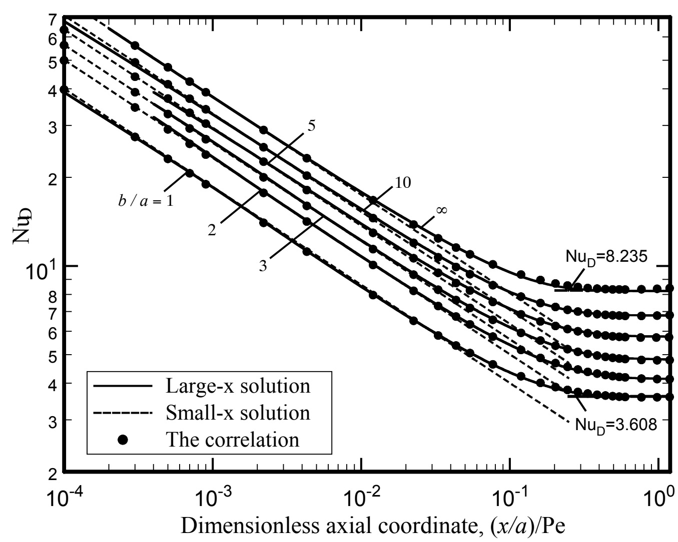

Figure 1. Computed data from small-x solution and function specification method compared with the above correlation.

The methodology used to acquire these data also applies to ducts having other cross section profiles. The application of this inverse technique to porous passages is interesting while the numerical computation is challenging.